Generation of high wave number fluctuations by external magnetic field perturbations in edge pedestal

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What are various physics issues in edge pedestal with external magnetic field perturbations (EMP)?
1. Magnetic field penetration across resonant surface and magnetic island generation?
2. Effect of EMP on plasma rotation, density and temperature?
3. Magnetic reconnection in ELM crash – important candidate?
4. Change in KBM spectrum with EMP?

“The effect of EMP on KBM Instability is investigated”
Contents:

- ELM types
- Physics models - linear and nonlinear simulations with and without EMP
- Nonlinear mode-mode coupling analysis on generation of high-k fluctuations by EMP:
- Results and Summary
- Low-n irregularities in high field side: a guess
Different ELM types

- At critical power $\rightarrow$ L-H mode transition $\rightarrow$ edge barrier
- High pressure gradient good for confinements $\rightarrow$ different ELMs types

$\alpha_c = q^2 R \beta / L_p$; $\beta = 8\pi p / B^2$, $p = n(T_i + T_e)$, $L_p^{-1} = -\partial_r \ln p$
Important parameter for ELM!!

- External magnetic field perturbations - RMP can suppress the giant type-I ELMs
- D-IIID with n=2-3 perturbations with various collisionality and shaping (Moyer et al, PoP 2005)
- Other tokamak experiments on ELMs mitigation by EMP have similar results

- JET with n=1, n=2 perturbations - Liang et al NF-2010
- MAST n=3 perturbations – Denner et al, NF-2012
- ASDEX-U with n=2 perturbations - Suttrop et al, PRL-11
  KSTAR with n=1 perturbations - Yun et al, PRL-11

1. Basic Ideas to mitigate ELMs by RMPs were:
   RMPs with sufficient magnitude would produce overlapping magnetic islands → greatly enhance the stochastic plasma transport → that reduces the pressure gradients and currents etc. the driving sources for ELMs instability
Various thoughts on ELMs

- Ideal BM with FLR – the KBM instability often used for explaining edge pedestal transport in H-mode plasmas (Snyder et al, PoP -01) KBM can limit the pressure gradient in PED (Dickinson et al, PPCF-09; PRL-11).

- PB instability is used to predict ELM onset and successfully predicted pedestal widths and heights (Snyder et al PoP 2009).

- “But not clear what’s the role of high-n KBM on ELM crash”

- Recently BOUT++ nonlinear MHD simulations with resistivity and hyper resistivity proposed the mechanisms for ELMs crash
1. Xi et al, PRL-14 suggests a nonlinear criterion for an ELM crash based on a phase space dynamical process i.e. phase coherence time \( \tau_{\phi-p}^c > \gamma_b^{-1} \) and importance of the initial profile of growth rate spectrum \( \gamma_b(k_\theta) \) in an ELM crash linear i.e. whether the result is ELM crash or turbulence.

2. Rhee et al, NF-14: the edge pedestal collapse due to abrupt stochastization due to nonlinear interaction between the adjacent ballooning modes (BM) and generation of tearing modes.

- ELMs mitigation by external magnetic perturbations in linear MHD and nonlinear RMHD simulations

1. 3D perturbations can substantially modify the local magnetics shear and reduce the critical pressure gradient for ideal BM \( \rightarrow \)
onset the KBM instability (micro-instabilities) at lower pressure gradient $\rightarrow$ this could halt the inward progression and suppressed the PB mode – Bird and Hegna, NF-13

2. Nonlinear RMHD simulations with $n=2$ perturbations: $n=2$ can couple with BM and drive additional higher- n micro tearing modes, which mitigate the ELM energy released - Becoulet et al, PRL-2014; IAEA-04

- It is widely believed that type-I ELMs largely occur at BM threshold boundary

$$\nabla \cdot J = \nabla \cdot J_{pol} + \nabla \cdot J_{*p} + \nabla || J || = 0$$

$$\omega_0^r = -\frac{1}{2} (1 + \tau_{i} K) k_y, \quad \gamma_0 = \sqrt{\varepsilon_n (1 + \tau_{i} K) \frac{k^2_y}{k^2_\perp} - \frac{2k^2_\parallel}{\beta_e} - \frac{J'_0 k_y k^2_y}{k^2_\perp} - \frac{(1 + \tau_{i} K)^2 k^2_y}{4} + \Lambda(\delta B)}$$

$$\alpha \geq \alpha_c, \quad \alpha = \beta q^2 R / L_p$$
Motivations: The shape of growth rate spectrum $\gamma(k_\theta)$ is important to whether the result is a crash or turbulence [4].

- Basic equations:

$$\frac{\partial}{\partial t} \nabla^2_\perp (\phi + \tau_i \tilde{p}) + \tilde{\varepsilon}_n \tau_i \nabla_y \tilde{p} + \nabla \nabla^2_\perp \tilde{A} = (\beta / 2)[\tilde{A}, \nabla^2_\perp \tilde{A}]$$

$$\frac{\partial \tilde{p}}{\partial t} - \frac{5 \tilde{\varepsilon}_n \tau_i}{3} \nabla_y \tilde{p} + \left(1 + \eta - \frac{5 \tilde{\varepsilon}_n}{3}\right) \nabla_y \phi - \frac{5}{3} \frac{\partial}{\partial t} \nabla^2_\perp (\phi + \tau_i \tilde{p}) = 0$$

$$\partial_t \tilde{A} + (2 / \beta) \nabla \phi = -[\phi, \tilde{A}]$$
We employ a two-step mode-mode coupling parametric process which enables us to evaluate contributions from all higher toroidal modes.

- We consider the external magnetic field and BM perturbations

\[
\tilde{A}_0 = A_0 \left[ \exp \left( i \left( \vec{k}_{0\perp} \cdot \vec{r}_0 + k_{0\parallel} \cdot \vec{r} \right) + c.c \right) \right],
\]

\[
\tilde{f} (\vec{r}, t) = \sum_{l=-\infty}^{l=\infty} \tilde{f}_l \exp \left[ i \left( \vec{k}_{\perp} +lk_{0\perp} \right) \cdot \vec{r} + i (k_{\parallel} + lk_{0\parallel}) z - i \omega t \right]
\]

- Here we take the external magnetic field perturbations are time independent and localized in the vicinity of the rational magnetic surface.

- This leads to infinite series
\[ i \varepsilon \phi = \Lambda_0 (\phi_{+1} - \phi_{-1}); \quad i \varepsilon_{\pm 1} \phi_{\pm 1} = \Lambda_0 (\pm \phi_{\pm 1} + \phi_{\pm 2}); \]

\[ i \varepsilon_{\pm 2} \phi_{\pm 2} = \Lambda_0 (\pm \phi_{\pm 1} + \phi_{\pm 3}); \quad \ldots \]

\[ \varepsilon = \hat{\omega} k^2_{\perp} + \frac{\tau_i (\tilde{\varepsilon}_n \hat{k}_{\theta} + \hat{\omega} k^2_{\perp}) \left[ (1 + \eta - 5\tilde{\varepsilon}_n / 3) \hat{k}_{\theta} - (5\tau_i / 3) \hat{\omega} k^2_{\perp} \right]}{\hat{\omega} (1 + 5\tau_i \hat{k}^2_{\perp} / 3) + 5\tau_i \tilde{\varepsilon}_n \hat{k}_{\theta} / 3} - \frac{2k_{||} k^2_{\perp} k_{||}}{\beta \hat{\omega}} \]

\[ \left( \frac{\Lambda_0^2}{i \varepsilon_{+1} + \Lambda_0^2 / (i \varepsilon_{+2} + \Lambda_0^2 / (i \varepsilon_{+3} + \cdots \infty))} + \frac{\Lambda_0^2}{i \varepsilon_{-1} + \Lambda_0^2 / (i \varepsilon_{-2} + \Lambda_0^2 / (i \varepsilon_{-3} + \cdots \infty))} \right) \tilde{\phi} = 0 \]

\[ \Lambda_0 = 2\hat{\varepsilon}_{||} \cdot (\vec{k} \times \vec{k}_0) k_{|| \pm 1} A_0 k^2_{\perp} / \hat{\omega} \]

- For \( k_{\perp} > k_{0\perp} \) (\( n >> n_0 \))

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\[ i\varepsilon + 2S = 0; \quad S = \Lambda_0^2 / (i\varepsilon + S) \]

\[ \varepsilon = \pm 2 |\Lambda_0| \]

\[ \Lambda_0 = 2\hat{e}_\parallel \cdot (\vec{k} \times \vec{k}_0)k_{||\pm 1}\tilde{A}_0k_\perp^2 / \omega \]

- Local and non-local with trial function \( \phi = (1 + \cos \theta) / \sqrt{3\pi} \)
  for BM (Coppi NF-79), the averaged over eigen-function

\[ \left\langle \tilde{A} \right\rangle = \int_{-\pi}^{\pi} (\phi^* \tilde{A} \phi) d\theta / \int_{0}^{2\pi} |\phi|^2 d\theta \]
- With FLR and external magnetic field perturbations \((\alpha_{\delta B} \neq 0)\), frequency and the growth rate of BM:

\[
\bar{\omega}_r = -\frac{\hat{k}_\theta}{2} \left(\alpha (1 + \eta) / \varepsilon_n\right)^{1/2}
\]

\[
\bar{\gamma} = \frac{1}{f^{1/2}} \sqrt{\left(\alpha \bar{g} - \bar{h} + \Lambda_{\delta B} \hat{k}_\theta \bar{f}\right) - (1 + \eta) \alpha \hat{k}_\theta^2 \bar{f} / 4\varepsilon_n}
\]

- New Feature: For \(\alpha < 1\) - the flat density and pressure profiles, a new mode at Alfvén time scale can be excited if

\[
\bar{\gamma} = \sqrt{\left(\Lambda_{\delta B} \hat{k}_\theta - 1\right)}; \quad \Lambda_{\delta B} \hat{k}_\theta = (4nq^2 / \varepsilon^{1/2})(\delta B^{ext} / B) > 1
\]
Physics picture for BM growth enhanced in $k_\theta$:

- EMP modulates the local magnetic field line in the vicinity of a rational surface.
- The coupling between the modulated wave, $\tilde{A}_0 = A_0 \exp(ik_{0\perp} \cdot r)$ and BM perturbations induces Maxwell stress as a nonlinear stress on the plasma via the interaction of $(A_0, A_{\pm1})$ in the vorticity equation.
- This induced force weakens the linear field line bending since the nonlinearly induced stress and the linear stress in field line bending are opposite sign for the BM.
- The nonlinear stress cancels the linear stress, which effectively stimulates BM drive nonlinearly.
Summary:

- Significant broadening in $\gamma(k\theta)$ spectrum takes place near BM threshold (i.e. $\alpha \leq 1$) with increasing EMFP.

- A nonlinear instability is found due to the coupling between Alfvén waves and EMPs.

- The BM stability boundary is strongly modified by the presence of EMPs, in particular, at small $\alpha$, where a stability window is predicted.

- The broadening of $\gamma(k\theta)$ spectrum with $\delta B$ can lead to high-$k\theta$ BM turbulence and enhance the turbulent transport in pedestal may mitigate an ELM crash.

- The modification of $\gamma(k\theta)$ profile may yield the reduction of the phase coherent time, leading to a possible quiet H-mode state [4].
Comments:

“Understanding of pedestal relaxation and the physics of transient outburst is still an open challenging topic to investigate”

Thank You